Workshop
Analysis, Geometry and PDE
in Honor of the 70th Birthday of Kenro Furutani
Leibniz Universität Hannover
October 18, 2019

10:00 - 10:40	Takeshi Hirai	On Theories of Representations and Characters
10:50 - 11:30	Ryszard Nest	Around the Functional Equation
11:30 - 11:50	Tea and Coffee	
11:50 - 12:30	Bernhard Gramsch	The Complex Homotopy Principle for Pseudodifferential Operators on Singular Spaces
12:30 – 14:00	Lunch Break	
14:00 - 14:40	Kazuaki Taira	An Index Formula for the Relative Hodge-Kodaira Theory
14:50 - 15:30	Maurice de Gosson	The Pancharatnam-Sjöqvist Phase Shift and the Conley-Zehnder Index
15:30 – 16:00	Tea and Coffee	
16:00 – 16:40	Chisato Iwasaki	On the Heat Kernel for Forms on the Heisenberg Group
16:50 – 17:30	Matthias Lesch	The KO-valued Spectral Flow for Skew-adjoint Fredholm Operators
19:00		Restaurant A'mura, Schlossstrasse 6, 30159 Hannover

Abstracts

The Pancharatnam-Sjöqvist Phase Shift and the Conley-Zehnder Index

Maurice de Gosson

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Abstract

Positive trace-class operators with unit trace ("density operators") represent the mixed states of quantum mechanics. In this talk we focus on the case where these operators have Gaussian Weyl symbols, and address the notion of relative phase shifts when these operators are acted upon by metaplectic isotopies. In particular we study the Pancharatnam-Sjöqvist phase shift which intervenes in quantum state reconstruction problems. The key ingredient in this study is the theory of the Conley-Zehnder index which is an intersection index closely related to the Leray-Maslov index. We take the opportunity to recall the definition and properties of the Conley-Zehnder index which originates from the theory of periodic Hamiltonian orbits.

The Complex Homotopy Principle for Pseudodifferential Operators on Singular Spaces

Bernhard Gramsch

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Abstract

In a finite dimensional setting the Oka principle is presented in [1] including substantial work from the period 2000-2010. We propose an extension of some of these results for holomorphic operator valued functions on

Stein manifolds. Let $H := H(X, M)$ resp. $C := C(X, M)$ be the set of holomorphic resp. continuous mappings from a holomorphy region X into the complex Fréchet manifold M; the statement (J) says that for certain choices of M for each $n = 0, 1, 2, \ldots$ the homotopy sets of H and C are isomorphic by deformation. In the case that M is the manifold of complex $(k \times k)$ - matrices of kernel dimension $< r$ $(1 < r < k - 1)$ (J) had been proved by Gromov (1989). If M is the group of invertible matrices the statement (J) is a well known result of Grauert (1958). Starting from [5,6] new extensions of (J) are given for some classes of operators also on singular (see [4]) manifolds. A motivation to consider the isomorphism (J) in the case of locally bounded (p-normed) operator algebras comes from [2], the theory of operator ideals (see [3]) and the work of Triebel on function spaces. Some problems for infinite dimensional Lie groups M are mentioned.

References.

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On Theories of Representations and Characters

Takeshi Hirai

Kyoto University

Abstract

The theory of characters and linear representations of finite groups begun with three papers of Frobenius [F53], [F54] in 1896 and [F56] in 1897. The theory of spin representations (or projective representations or multivalued representations) of finite groups begun with three papers of Schur [S4] in 1904, [S10] in 1907 and [S16] in 1911. Schur also studied the construction and the explicit determination of characters of the motion groups $SO(n)$ (after the infinitesimal results for Lie algebra $\mathfrak{so}(n, \boldsymbol{R})$ due to É. Cartan) in three papers [S51]–[S53] in 1924, where he applies the results of Huewitz [Hur1897] on invariant measures on $SO(n)$. Before [S53] was published, Weyl took communication with Schur about his finding that, he invented an integration formula applicable for invariant functions on the group, and using it one can obtain a unified character formula in a form of fractionals with the same kind of formulas for the numerator and the denominator (all of them are called Weyl's ones today). With encouragements of Schur, Weyl finished his great work [W68].

— Not referring anything in between —

In 1962, I begun my work with the classification of irreducible representations, calculation of their characters and so on, of general Lorentz groups $SO_0(n, 1)$, $n \geq 5$, in [Hira1]–[Hira4] in '62, '65, '66, and similar things of special unitary group $SU(p, 1)$, $p \geq 2$ in [Hira5] in '66. In succeeding years, working principally on representations of semisimple Lie groups, I arrived to [Hir1981]. —— I remark that Thoma's work [Tho] in 1964 opened another world by giving all the characters of factor representations of the infinite symmetric group $\mathfrak{S}_\infty$ as extremal positive definite invariant functions on $\mathfrak{S}_\infty$. Later this work stimulates many studies in various directions. Also I worked on several subjects surrounding infinite (locally finite) discrete groups together with N. Obata and A. Hora. Recently I am working with Hora on spin characters of infinite complex reflection groups.

To show the interrelations of all these things, we list up many references here. If time permits I would like to explain a little more in detail about [HHo2018].

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On the Heat Kernel for Forms on the Heisenberg Group

Chisato Iwasaki

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Abstract

I will talk about a representation of the heat kernel for forms on the Heisenberg group. This representation is obtained by using the fundamental solutions of a series of systems of ordinary differential equations of second order.

This study is one of generalization of the result which we have obtained for one-form on the Heisenberg group. This is a joint work with W. Bauer and K. Furutani.

The KO–valued Spectral Flow for Skew-adjoint Fredholm Operators

Matthias Lesch

University of Bonn

Abstract

We give a comprehensive treatment of a 'Clifford module flow' along paths in the skew-adjoint Fredholm operators on a real Hilbert space that takes values in $KO_*(\mathbb{R})$ via the Clifford index of Atiyah–Bott– Shapiro. We develop its properties for both bounded and unbounded skew-adjoint operators including an axiomatic characterization. Our constructions and approach are motivated by the principle that

$$\text{spectral flow} = \text{Fredholm index}.$$

That is, we show how KO–valued spectral flow relates to a KO–valued index by proving Robbin–Salamon type result. The Kasparov product is also used to establish a spectral flow = Fredholm index result at the level of bivariant K-theory. We explain how our results incorporate previous applications of $\mathbb{Z}/2\mathbb{Z}$-valued spectral flow in the study of topological phases of matter.

Around the Functional Equation

Ryszard Nest

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Abstract

The functional equation for the Riemann zeta function is based on analysis of asymptotic behaviour for $t \approx 0$ of expression such as $\mathrm{Tr}(\exp(-tD^2))$, where D is, say, an elliptic operator on a smooth closed manifold M. In particular, it depends heavily on the the fact that expressions like $\mathrm{Tr}(\exp(-tD^2))$ have Mellin transform which is holomorphic on a subspace of the complex plane of the form $\mathrm{Re}(z) > C$, which is a consequence of finite dimensionality of M. We will construct an analogue of the meromorphic extension of the Riemann zeta function and prove the corresponding functional equation in the infinite dimensional limit case. We will sketch some work in progress which give applications of these constructions to local index formulas for operators associated to infinite dimensional physical systems.

An Index Formula for the Relative Hodge-Kodaira Theory

Kazuaki Taira

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Abstract

The purpose of my talk is to give an analytic proof of an index formula for the relative de Rham cohomology groups which may be considered as a generalization of the classical Hodge-Kodaira theory for the absolute de Rham cohomology groups. The crucial point in the proof is how to find an operator of the matrix form for which an index formula holds true in the framework of spaces of currents. In deriving our index formula of Agranovič-Dynin type, the theory of generalized harmonic forms satisfying an interior boundary condition plays a fundamental role. Our index formula is an analytical understanding of the Gauss-Bonnet-Chern theorem for two manifolds of even dimension.