Leibniz Universität Hannover

Book of Abstracts

Workshop „Analysis and PDE“

October 8 – 10, 2018

Workshop
Analysis and PDE 2018

Schedule

Hour	Monday	Tuesday	Wednesday	Hour (Wed)
09:00 - 09:40	Reichel	Niethammer	Gallagher	09:00 - 09:40
09:50 - 10:30	Mingione	Malek	Nicaise	09:50 - 10:30
10:30 - 11:00	Tea and Coffee	Tea and Coffee	Tea and Coffee	10:30 - 11:00
11:00 - 11:40	Post	Abels	King	11:00 - 11:40
11:50 - 12:15	Froehly	Frey	Stinner	11:50 - 12:15
	Lunch Break	Lunch Break	Lunch Break	
14:00 - 14:40	Rizzi	Barilari	Schwarzacher	14:00 - 14:40
14:50 - 15:15	Fritzsch	Grong	Skalak	14:50 - 15:15
15:20 - 15:45	Weich	Hagger	Tea and Coffee	15:20 - 15:50
15:50 - 16:20	Tea and Coffee	Tea and Coffee	Nobili	15:50 - 16:15
16:20 - 16:45	Krietenstein	Molino	Necasova	16:20 - 17:00
16:50 - 17:30	Vespri	Baudoin		

Sharp Interface Limit for Two-Phase Flows of Incompressible Fluids

Helmut Abels

Universität Regensburg
helmut.abels@mathematik.uni-regensburg.de

Abstract

We consider the sharp interface limit of a coupled Stokes/Allen-Cahn system, when a parameter $\varepsilon > 0$ that is proportional to the thickness of the diffuse interface tends to zero, in a two dimensional bounded domain. For sufficiently small times we prove convergence of the solutions of the Stokes/Allen-Cahn system to solutions of a sharp interface model, where the interface evolution is given by the mean curvature equation with an additional convection term coupled to a two-phase Stokes system with an additional contribution to the stress tensor, which describes the capillary stress. To this end we construct a suitable approximation of the solution of the Stokes/Allen-Cahn system, using three levels of the terms in the formally matched asymptotic calculations, and estimate the difference with the aid of a suitable refinement of a spectral estimate of the linearized Allen-Cahn operator. Moreover, we will discuss recent extensions of this results e.g. to the Navier-Stokes/Allen-Cahn system or the Stokes/Cahn-Hilliard system.

On the Brunn-Minkovski inequality in sub-Riemannian geometry

Davide Barilari

Université Paris Diderot
davide.barilari@imj-prg.fr

Abstract

In a celebrated paper of Cordero-Erasquin-McCann-Schmuckenschläger, the authors extend to the case of Riemannian manifolds some classical functional and geometrical interpolation inequalities in the Euclidean setting, using optimal transport techniques.

In particular these results imply a "geodesic" version of the classical Brunn-Minkowski inequality in the Euclidean space. In this talk, we discuss the validity of such results in the case of sub-Riemannian geometry. If time allows we will discuss a new sub-Riemannian Bakry-Emery type concept of sectional and Ricci curvature, to take into account the ambient measure in comparison-type results. [Joint work with Luca Rizzi]

Uniform sub-Laplacian comparison theorems on Sasakian sub-Riemannian manifolds

Fabrice Baudoin

University of Connecticut
fabrice.baudoin@uconn.edu

Abstract

We will present several results about sharp sub-Laplacian comparison for a family of metrics approximating the sub-Riemannian one. As a result, we will discuss measure contraction properties of Sasakian manifolds. This is joint work with Erlend Grong, Kazumasa Kuwada and Anton Thalmaier.

Paradifferential and paracontrolled calculus in rough settings

Dorothee Frey

Delft University of Technology
d.frey@tudelft.nl

Abstract

The paradifferential calculus plays an important roles in PDEs, in particular in the treatment of nonlinearities in Sobolev or Besov spaces. In recent years, it has found

major applications in the rough path theory. Gubinelli, Imkeller and Perkowski have established a so-called paracontrolled calculus as an alternative approach to Hairers regularity structures in the context of singular stochastic PDEs.

We will discuss the basic principles of a paracontrolled calculus for singular stochastic PDEs, and show how it can be adapted to non-smooth settings where no Fourier transform is available.

Decomposable Monopoles, Clusters and a Conjecture of Sen

Karsten Fritzsch

Leibniz University Hannover
k.fritzsch@math.uni-hannover.de

Abstract

The moduli space of (non-abelian) magnetic monopoles is, for fixed charge k, a smooth, complete and non-compact hyperKähler manifold of dimension $4k$. The asymptotic behaviour of the metric has been studied in various special cases. Apart from the intrinsic interest in understanding the asymptotic behaviour completely, this is also important for the study of L^2 -harmonic forms, which are the subject of Sen's Conjecture.

I will report on joint work with C. Kottke and M. Singer on the construction of a geometric compactification of the moduli space. In particular, I will describe the construction of a cover in terms of "clusters of monopoles" and how this will lead to a proof of the coprime case of Sen's Conjecture.

Limiting absorption principle and spectral asymptotics for waveguides with singular perturbations

André Froehly

Leibniz University Hannover
andre.froehly@math.uni-hannover.de

Abstract

We consider a general second-order elliptic differential operator on a domain with a cylindrical end. We impose Dirichlet conditions on the boundary with the exception of a small set, where we impose Neumann boundary conditions. We prove a corresponding limiting absorption principle for the resolvent and show an asymptotic formula for the resonances in the case that the Neumann window shrinks to a point.

On the behaviour at blow-up of solutions to the incompressible 3D Navier-Stokes equations

Isabelle Gallagher

Université Paris Diderot
Isabelle.Gallagher@ens.fr

Abstract

The question of blow up in finite time of solutions to the incompressible 3D Navier-Stokes equations is a long-standing open problem. In this talk we shall review some results describing or excluding some blow-up scenarios.

Sub-Laplacians on forms and cohomology

Erlend Grong

Université Paris-Saclay
erlend.grong@u-psud.fr

Abstract

For any sub-Riemannian structure, there is a unique corresponding second order operator on functions which is symmetric with respect to some given volume form. We will call this operator the sub-Laplacian. We discuss the challenges, results and applications that come from attempting to generalize this operator to differential forms. We will mainly be concerned with operators which has a Weitzenböck decomposition, that is, can be constructed from the Hessian of an affine connection and a zero order differential operator. As one applications, we get results related to cohomology of totally geodesic foliations.

A Product Expansion for Toeplitz Operators

Raffael Hagger

Leibniz University Hannover
raffael.hagger@math.uni-hannover.de

Abstract

Consider two Toeplitz operators T_f and T_g on the Hardy Space H^2. By a well-known result of Brown and Halmos, their product T_fT_g is again a Toeplitz operator if and only if either $\bar{f}$ or g is holomorphic, i.e. only in the trivial cases. For Toeplitz operators on the Fock space the situation is different. For instance, if f and g are polynomials in z and $\bar{z}$, then

$$T_fT_g = \sum_{j=0}^{\infty} \frac{(-1)^j}{j!} T_{(\partial^j f)(\bar{\partial}^j g)}. \tag{1}$$

As this sum is finite, the right-hand side is again a Toeplitz operator. However, (1) clearly does not hold for all C^∞-functions f and g. In fact, for most functions (1) does not even converge. By introducing a semi-classical parameter, we will view (1) as an asymptotic expansion and give sufficient conditions for this product expansion to hold. The result then has a direct application to Toeplitz quantization.

Analysis of moving boundary problems associated with biological tissue growth

John King

University of Nottingham
john.king@nottingham.ac.uk

Abstract

Some macroscale (PDE) formulations motivated by tissue-engineering applications will be described and some results of their asymptotic analysis described.

Bounded H^∞-calculus for a Degenerate Elliptic Boundary Value Problem

Thorben Krietenstein

Leibniz University Hannover
krietenstein@math.uni-hannover.de

Abstract

We consider a strongly elliptic second order operator A together with a degenerate boundary operator T of the form $T = \varphi_0\gamma_0 + \varphi_1\gamma_1$, where γ_0 and γ_1 denote the evaluation of a function and its exterior normal derivative, respectively, at the boundary, and $\varphi_1 \geq 0$, $\varphi_0 + \varphi_1 > 0$. We show that the realization A_T of A in $L_p(\Omega)$ has a bounded H^∞-calculus whenever Ω is a compact manifold with boundary or $\Omega = \mathbb{R}^n_+$.

PDE analysis for a class of thermodynamically compatible viscoelastic rate-type fluids with stress-diffusion

Josef Málek

Charles University, Prague
malek@karlin.mff.cuni.cz

Abstract

We establish the long-time existence of large-data weak solutions to a system of nonlinear partial differential equations. The system of interest governs the motion of non-Newtonian fluids described by a simplified viscoelastic rate-type model with a stress-diffusion term. The simplified model shares many qualitative features with more complex viscoelastic rate-type models that are frequently used in the modeling of fluids with complicated microstructure.

As such, the simplified model provides important preliminary insight into the mathematical properties of these more complex and practically relevant models of non-Newtonian fluids. The simplified model that is analyzed from the mathematical perspective is shown to be thermodynamically consistent.

We present the results regarding existence of globalintime weak solutions for any finite energy initial data both for incompressible and compressible fluids.

The lecture is based on the results achieved together with M. Bulíček, E. Feireisl, V. Průša and E. Süli.

Recent progresses in nonlinear potential theory

Giuseppe Mingione

University of Parma
giuseppe.mingione@unipr.it

Abstract

Nonlinear potential theory is actually a part of regularity theory that aims at reproducing, for nonlinear equations, the classical results from linear potential theory about fine properties of harmonic functions and integrability properties of solutions. I will present a number of recent results, aimed at giving sharp pointwise bounds for solutions to nonlinear elliptic and parabolic equations in terms of linear and nonlinear potentials.

The Horizontal Einstein Property for H-Type sub-Riemannian Manifolds

Gianmarco Molino

University of Connecticut
gianmarco.molino@uconn.edu

Abstract

We generalize the notion of H-type sub-Riemannian manifolds introduced by Baudoin and Kim, and then introduce a notion of parallel Clifford structure related to a recent work of Moroianu and Semmelmann. On those structures, we prove an Einstein property for the horizontal distribution using ideas from Ishihara's work on hyper-Kähler and quaternionic Kähler manifolds. This is a joint work with F. Baudoin, E. Grong, and L. Rizzi.

On the Problem of the Motion of a Rigid Body with a Cavity Filled with Compressible Viscous Fluid

Šárka Nečasová

Academy of Sciences of the Czech Republic, Prague
matus@math.cas.cz

Abstract

We study the motion of the system, $\mathcal{S}$, constituted by a rigid body, $\mathcal{B}$, containing in its interior a viscous compressible fluid, and moving in absence of external forces. Our main objective is to characterize the long time behavior of the coupled system body-fluid. Under suitable assumptions on the "mass distribution" of $\mathcal{S}$, and for sufficiently "small" Mach number and initial data, we show that every corresponding motion (in a suitable regularity class) must tend to a steady state where the fluid is at rest with respect to $\mathcal{B}$. Moreover, $\mathcal{S}$, as a whole, performs a uniform rotation around an axis parallel to the (constant) angular momentum of $\mathcal{S}$, and passing through its center of mass. It is a joint work with G. P. Galdi and V. Mácha.

References

[1] G. P. Galdi V. Mácha Š. Nečasová: *On the Motion of a Body with a Cavity Filled with Compressible Fluid*, Submitted.

Stabilization of a Drude-vacuum model

Serge Nicaise

University of Valenciennes and Hainaut-Cambresis
snicaise@univ-valenciennes.fr

Abstract

We analyze the stability of a dispersive medium immersed in vacuum (with Silver-Müller boundary condition in the exterior boundary) or vice versa. The dispersive medium model corresponds to the coupling between Maxwell's system and a first order ordinary differential equation (of parabolic type). For a dispersive medium coupled with vacuum, the ordinary differential equation will be set in a subset of the full domain. We show that this model is well-posed and is strongly stable in a closed subspace of the energy space. We further identify some sufficient conditions that guarantee the exponential or polynomial decay of the associated energy in this subspace.

Self-similarity in Smoluchowski's coagulation equation

Barbara Niethammer

Universität Bonn
niethammer@iam.uni-bonn.de

Abstract

Smoluchowski's classical mean-field model for coagulation is used to describe cluster formation and growth in a large variety of applications. A question of particular relevance is the so-called scaling hypothesis, which suggests that the long-time behaviour is universal and described by self-similar solutions or traveling waves respectively. This issue is well understood for few exactly solvable cases, but in the general case many questions are still completely open. I will give an overview of the results that have been obtained in the last decade and explain why we expect that the scaling hypothesis is not true in general. (based on joint work with Marco Bonacini, Michael Herrmann and Juan Velazquez)

A maximal regularity estimate for the non-stationary Stokes equation in the strip

Camilla Nobili

Universität Hamburg
camilla.nobili@uni-hamburg.de

Abstract

In this talk we present a new maximal regularity estimate for the non-stationary Stokes equation with no-slip boundary condition in the strip. This estimate will be crucial for estimating the average upward heat transport in the Rayleigh-Bénard convection problem.

Norm resolvent convergence of Laplacians on various singular perturbations

Olaf Post

University of Trier
olaf.post@uni-trier.de

Abstract

In this talk we apply a general convergence scheme which generalises norm resolvent convergence to the case of parameter-dependent spaces. We briefly introduce the general convergence scheme and its consequences such as convergence of the semi-groups or the spectrum and show how it can be applied in various situations such as manifolds shrinking towards a metric graph, manifolds with many little holes removed or manifolds with many little handles added.

Equilibrium distributions in the Born-Infeld electrostatic theory

Wolfgang Reichel

Karlsruhe Institute of Technology
Wolfgang.Reichel@kit.edu

Abstract

This talk reports on joint work with D. Bonheure (Univ. Libre de Bruxelles), P. D'Avenia (Politecnico di Bari), and A. Pomponio (Politecnico di Bari). Within the Born-Infeld electrostatic theory, the electrostatic potential generated by a probability measure ρ on the boundary $\partial\Omega$ of a domain $\Omega \subset \mathbb{R}^3$ is the unique minimizer ϕ_ρ of the Born-Infeld electrostatic action (with $b > 0$ a positive constant)

$$\mathcal{I}_\rho(\phi) = \int_{\mathbb{R}^3} \left(b^2 - b\sqrt{b^2 - |\nabla\phi|^2} \right) dx - \oint_{\partial\Omega} \phi d\rho.$$

Here ϕ runs through the set $\mathcal{X}$ of all $D^{1,2}(\mathbb{R}^3)$ functions with Lipschitz-constant less or equal b. For each electrostatic potential ϕ_ρ we can consider the Born-Infeld electrostatic energy $\mathcal{E}$ given by

$$\mathcal{E}(\phi_\rho) := -\mathcal{I}_\rho(\phi_\rho).$$

Among all possible charge distributions one can search for those distributions ρ^*, which create least-energy potentials. Such a minimizing distribution ρ^* (provided it exists) is called *equilibrium distribution*. Its corresponding potential ϕ_{ρ^*} is called an *equilibrium potential*. The main purpose of this talk is to provide the existence and the properties of the equilibrium distribution and the equilibrium potential. The four most important results are:

(i) equilibrium distributions exist;

(ii) the equilibrium potential ϕ_{ρ^*} is unique and takes a constant value λ^* in $\overline{\Omega}$;

(iii) if $\partial\Omega \in C^{2,\alpha}$ then also the equilibrium distribution ρ^* is unique and the equilibrium potential is a weak solution of the Euler-Lagrange equation associated with $\mathcal{I}_\rho$;

(iv) within the class of $C^{2,\alpha}$ domains, the ball is the unique member with equilibrium distribution being a constant multiple of the surface measure.

Similar results can be achieved for approximated electrostatic actions, where the action integrand $b^2 - b\sqrt{b^2 - |\xi|^2}$ (with ξ a placeholder for $\nabla\phi$) is replaced by its Taylor-polynomial.

Weyl's law for singular Riemannian manifolds

Luca Rizzi

Institut Fourier, Grenoble
luca.rizzi@univ-grenoble-alpes.fr

Abstract

We study the asymptotic growth of the eigenvalues of the Laplace-Beltrami operator on singular Riemannian manifolds, where all geometrical invariants appearing in classical spectral asymptotics are unbounded, and the total volume can be infinite. Under suitable assumptions, we prove that the leading term of the Weyl's asymptotics contains information on the singularity, i.e. its Minkowski dimension and its regularized measure. We apply our results to a suitable class of almost-Riemannian structures. A key tool in the proof is a new heat trace estimate with universal remainder for Riemannian manifolds, which is of independent interest. Work in collaboration with Y. Chitour and D. Prandi.

On compressible fluids interacting with a linear-elastic Koiter shell

Sebastian Schwarzacher

Universität Bonn
schwarz@iam.uni-bonn.de

Abstract

The lecture is about the motion of a viscous incompressible fluid in three dimensions interacting with a flexible shell. The shell constitutes a moving part of the boundary of the physical domain. Its deformation is modeled by a linearized version of Koiter's elastic energy. We discuss the existence of weak solutions to the corresponding system of PDEs provided the adiabatic exponent satisfies $\gamma > \frac{12}{7}$ ($\gamma > 1$ in two dimensions). It will be explained that a weak solution exists until the moving boundary approaches a self-intersection. This is a joint work with D. Breit (Heriot-Watt Univ. Edinburgh).

Some regularity results for the Navier-Stokes equations

Zdeněk Skalák

Czech Technical University in Prague
zdenek.skalak@cvut.cz

Abstract

We show several results concerning the conditional regularity for the Navier-Stokes equations in the entire three dimensional space. Supposing that $u = (u_1, u_2, u_3)$ is a solution, we mainly focus on the cases, where the additional conditions are imposed on $\partial_1 u_3$, ∇u_3 and $\partial_3 u$. We comment on recent results from the literature and then prove some improvements and extensions.

References

[1] C. Cao, *Sufficient conditions for the regularity to the 3D Navier-Stokes equations*, Discrete Contin. Dyn. Syst. **26** (2010), 1141–1151.

[2] I. Kukavica, M. Ziane, *Navier-Stokes equations with regularity in one direction*, J. Math. Phys. **48** (2007), 10 pp.

[3] Z. Zhang, *A improved regularity criterion for the Navier-Stokes equations in terms of one directional derivative of the velocity field*, Bull. Math. Sci. **8** (2018), 33–47.

An unusual critical mass phenomenon for a quasilinear chemotaxis system

Christian Stinner

Technische Universität Darmstadt
stinner@mathematik.tu-darmstadt.de

Abstract

For the minimal fully parabolic Keller-Segel system it is well-known that in dimension 2 there is a critical mass separating between boundedness of all classical solutions for subcritical mass and the existence of blow-up solutions for supercritical mass. In addition the blow-up for supercritical masses is known to occur in finite time. This type of critical

mass has also been shown for some other variants of the Keller-Segel system and seems to be the generic type for this class of systems.

However, for a quasilinear fully parabolic Keller-Segel system we establish a different type of critical mass. Namely, for any mass all classical solutions are global and there is a critical mass separating between boundedness of all solutions (for subcritical mass) and existence of solutions blowing up in infinite time (for supercritical mass). To the best of our knowledge, this type of critical mass had not been observed before in a Keller-Segel system with direct signal production. Our proof relies on an appropriate use of a Liapunov functional and the adaptation of methods from the minimal Keller-Segel system to the current setting.

This talk is based on a joint work with T. Cieślak.

Sharp propagation of the support of solutions to anisotropic degenerate evolution equations

Vincenzo Vespri

University of Florence
vincenzo.vespri@unifi.it

Abstract

In this talk, I will first discuss about anisotropic operators: what is it known, the (too many) open questions, the regularity for operators with smooth coefficients. Then we will speak about $L^1 - L^\infty$ estimates and we will apply these estimates to give a bound to the growth of the support of the solution. Lastly we will show the optimality of these estimates.

Dynamical resonances and spectral geometry

Tobias Weich

Paderborn University
tobias.weich@upb.de

Abstract

Dynamical (Ruelle-Pollicott) resonances have their origin in the study of chaotic dynamical systems. They were originally introduced by David Ruelle and Mark Pollicott in the 80s to study the convergence of chaotic dynamical systems towards equilibrium. During the last years a new spectral theoretic approach to dynamical resonances has been developed and thanks to these new techniques they became an interesting object in spectral geometry and topology.

In this talk I will introduce the notion of dynamical resonances for geodesic flows and review some recent results that demonstrate the interest of these resonances for spectral geometry.